Pricing European Options by Analytical Methods with Applications in Fidelity and Access Banks Share Prices.

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Abstract
This paper considered the framework of Black-Scholes model of European options on share price of Fidelity, Access and their future merged banks which gave closed form prices of Call and Put option prices with variations of maturity dates, affecting real life changes for capital markets. More so, the share price of the independent banks was used to form a system of linear equations; which gave three different solutions according to the banks under considerations whose solutions represents different outcomes for share prices to analyze and predict for the purpose of investment plans for capital markets by means of analytical method. Finally, we state and prove a proposition on solution of system of linear equations to show that our share price follows exponential series over time and obey some physical laws arising from financial markets. This paper offered here has profound inference for future studies of option prices and may one day help solve the problems of option traders and society at large.

Keywords: *European options, Share prices, Stochastic Analysis, Fidelity and Access Banks.*

1.1 Introduction

This paper is all about option pricing and some of its intricacies in financial markets as it explores the changes of option (values) as a function of time. Over time, investors have been trying their best to gain ground in stock market but could not discover mathematical model that predict price fluctuation not until some group of economists Fischer Black, Myron Scholes, and Robert .C. Merton, came up with a flexible and powerful mathematical model for financial markets based on certain assumptions Black-Scholes (BS) (1973). So this financial model could help options traders to properly price options. Hence ,Black-Scholes is a pricing model used to determine the fair price or theoretical value for a call or a put option based on five variables namely: volatility, underlying stock price, time, exercise price, and risk-free rate. Therefore, an option is a tool whose worth is derived from the principal asset which is otherwise known as financial derivative. This type of derivative does have anything in common with mathematical meaning of derivative. In other hand, an option on underlying asset is a business between parties who come together to agree on either buying or selling an underlying asset at a determined strike price in the future for a fixed price, Amadi et al.(2020) The Black-Scholes formula is a mathematical model to calculate the price of put and call options. Since put and call options are distinctly different, there are two formulas, which account for each option. Call options give the option holder the right to buy the underlying stock for an agreed upon price anytime between today and upon expiration. Traders that believe the underlying stock will go up over time buy these call options in the hopes of making money. On the flip-side, put options give the option holder the right to sell the underlying stock for an agreed upon price anytime between today and upon expiration. Traders that think a stock is going to go down can buy these put options in the hopes of making money if the stock goes does.

Many scholars have used Black-Scholes Equation in different approaches. For instance; Amadi et al.(2020) considered Black-Scholes partial differential equation on stock market prices for both analytic and numerical. In another scenario Shin and Kim (2016) focused on the Black-Scholes terminal value problem and provided its solutions through the Laplace transform. More so,Fadugba et al.(2023) proposed a framework based on the celebrated transform of Mellin type (MT) for the analytic solution of the Black-Scholes-Merton European Power Put Option Model (BSMEPPOM) on Dividend Yield (DY) with Modified-Log-Power Payoff Function (MLPPF) under the geometric Brownian motion. In the same vein, Babasola et al.(20080 analyzed BS formula for the valuation of European options; hermit polynomials were applied. They concluded that BS formula can easily be achieved devoid of the use of partial differential equation. In the work of Rodryo and Mamon(2006), time varying factor were incorporated in the explicit formula for different aspect of options with the aim of providing exact solution for dividend paying equity of option. In considering the stability of stock market price of stochastic model, Osu (2010) applied Crank-Nicolson numerical scheme to BS model. The results showed stock prices being stable and its increasing rate of stock shares was obtained. Not quite long, Nwobi et al.(2019) investigated the variation of stock market price using BS PDE. The convergence to equilibrium of growth rate and sufficient conditions for stability was achieved. Lots of scholars has written extensively to mention but a few:Fadugba and Nwozo(2013) and Osu and Okoroafor (2007) etc.

However, the major issues of investors or owners of corporations is the inability to take an appropriate decisions when using Black-Scholes model of option pricing with system of linear equations in analyzing share prices. These issues may have arisen due to the formulation of problem or inability to add more models to the study or inability to understand the analytical solutions as to interpret accurately to the real world market. These discrepancies may lead to models not predicting closed form solutions for European options or from or systems of linear equation under study for proper decision making; which may not be to the best interest of option traders or investors or management of Fidelity and Access banks. In order to tackle the above problem we impose European options, system of linear equations for prediction of share prices of Fidelity, Access , their future merging according to Osu et al.(2019).

This paper considered the framework of Black-Scholes model of European options on share price of Fidelity, Access and their future merged banks which gave closed form prices of Call and Put option prices with variations of maturity dates, average share prices as well as their respective standard deviations affecting real life changes for capital markets. From the share price analysis the growth rates of each bank were considered .More so, the share price of the independent banks were used to form a system of linear equations; which gave three different solutions according to the banks under considerations whose solutions represents different outcomes for share prices to analysis and prediction. To this end, we state and prove a proposition on solution of system of linear equations to show that our share price follows exponential series over time and obey some physical laws arising from financial markets. The above concepts are novelties of paper which was not considered by previous efforts such as Amadi (2020) and Osu et al. (2019) etc.

This study is arranged as follows: Section presents Mathematical framework, Analysis of results are seen in 3.1, Discussion of findings are seen in Section 4.1 and the paper is concluded in Section 5.1.

2.1 Mathematical Framework

In this paper, we review all relevant stochastic methods that would help in achieving the derivation. Firstly, our focus in Sub-Section 2.1.1 stochastic processes that would lead to stochastic differential equation, which gave rise to Black Scholes model of option as well as system of linear equations.

2.1.1 Stochastic Processes

It can also be seen as a statistical event that evolves time in accordance to probabilistic laws. Mathematically, a stochastic process may be defined as a collection of random variables which are ordered in time and defines at a set of time points which may be continuous or discrete.

2.1.2 Stochastic Differential Equations

Here, consider a market where the underlying asset price v , $0 \le t \le T$ on a complete probability space $(\Omega, f, \wp)$ is governed by the following stochastic differential equation:

$$dS(t) = \alpha S(t)dt + \sigma dw(t).\ 0 < v < \infty \ . \tag{1.1}$$

Theorem 2.1: (Ito's formula) Let $(\Omega, \beta, \alpha, \mathrm{F}(\beta)$ be a filtered probability space $X = \{X, t \ge 0\}$ be an adaptive stochastic process on $(\Omega, \beta, \alpha, \mathrm{F}(\beta)$ possessing a quadratic variation(X) with SDE defined as:

$$dX(t) = g(t, X(t))dt + f(t, X(t))dW(t)$$

$t \in \Re$ and for $u = u(t, X(t) \in C^{1\times 2}(\Pi \times \square))$

$$du(t, X(t)) = \left\{\frac{\partial u}{\partial t} + g\frac{\partial u}{\partial x} + \frac{1}{2}f^2\frac{\partial^2 u}{\partial x^2}\right\}d\tau + f\frac{\partial u}{\partial x}dW(t)$$

Adopting theorem 2.1 comfortably solves the SDE in (3/1) with a given solution below:

$$S(t) = S_0\ \exp\left\{\sigma dW(t) + \left(\alpha - \frac{1}{2}\sigma^2\right)t\right\}, \forall t \in [0,1] \tag{1.2}$$

2.2 The Black-Scholes Model

The Black-Scholes model is made up of on seven assumptions:

The asset price has characteristics of a Brownian motion with μ and σ as constants; the transaction costs or taxes are not allowed; the entire securities are absolutely divisible; dividend is not permitted during the period of the derivatives; unacceptable of riskless arbitrage opportunities; the security trading is constant; the option is exercised at the time of expiry for both call and put options.

In mathematical finance, an arbitrage argument show that any derivative $V(\mathrm{S},t)$ written on v must satisfy the partial differential equation of the form of option pricing; hence we have the following:
:

$$\frac{\partial V(\mathrm{S},t)}{\partial t}+\frac{1}{2}\sigma^2 S^2\frac{\partial^2 V(\mathrm{S},t)}{\partial S^2}+rS\frac{\partial V(\mathrm{S},t)}{\partial S}-rV(\mathrm{S},t)=0 \quad . \tag{1.3}$$

Where r represents interest rate , σ represents volatility of the underlying assets and t represents time of maturity.
With boundary conditions:

$$V(\mathrm{S},t)\to\infty \; as\, S\to\infty \; on\; [0,T). \; . \tag{1.4}$$

$$V(\mathrm{S},t)\to 0 \;\; as \;\; S\to 0 \; on[0,T). \; . \tag{1.5}$$

And final time condition given by :

$$V(S_T,T)=(S_T-k)^+ = f(S_T) \; on \; [0,\infty]. \tag{1.6}$$

Equation (1.5) is the value of asset is worthless when the stock price is zero, Amadi et al.(2024). The details of the above option model can be expressly be found in the following books: Black-Scholes (1973) and Fadugba et al.(2012) and Hull (2012) etc.
To eliminate the price process in (1.3- 1.6) slightly gives the Black-Scholes analytic formula for the prices of European call option is given as follows

$$\left.\begin{aligned} C &= SN(d_1)-Ke^{-rt}N(d_2) \\ d_1 &= \frac{\ln\left(\frac{S}{K}\right)+\left(r+\frac{\sigma^2}{2}\right)\tau}{\sigma\sqrt{\tau}} \\ d_2 &= d_1-\sigma\sqrt{\tau} \end{aligned}\right\} \tag{1.7}$$

where C is Price of a call option, S is price of underlying asset, K is the strike price, r is the riskless rate , T is time to maturity, σ^2 is variance of underlying asset, σ is standard deviation of the (generally referred to as volatility) underlying asset, and N is the cumulative normal distribution.
Similarly Black-Scholes analytic formula for the prices of European Put option is given as follows

$$\left.\begin{aligned} P &= SN(d_1)-Ke^{-rt}N(d_2) \\ d_1 &= \frac{\ln\left(\frac{S}{K}\right)+\left(r+\frac{\sigma^2}{2}\right)\tau}{\sigma\sqrt{\tau}} \\ d_2 &= d_1-\sigma\sqrt{\tau} \end{aligned}\right\} \tag{1.8}$$

where P is the price of a put option and the meaning of other parameters remain the same as in (1.7) Hull (2012).

2.3 Problem Formulation with System of Linear Equations

let $S_i(\mathrm{t})$ $\left(i=1,2,...,N,t=1,2,...,n\right)$ be the of yearly share price data selected years at time t be defined as three state matrices processes. Let $N\times n$ data matrix associated with $S_i(t)$ *be* X_{it} . We consider $t=1,2,3$ as row vector which multiplies each independent matrices to obtain payment changes of Fidelity, Access and their future merging hence, we have an estimates of the matrices as follows . However, for $k=3$ is an estimate of the matrices below.

$$FIDELITY-BANK(\mathrm{FB})=\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \tag{1.8}$$

$$ACCESS-BANK(\mathrm{AB})=\begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix} \tag{1.9}$$

$$FIDELITY-ACCESS-BANK-MERGED(\mathrm{FBM})=\begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{pmatrix} \tag{1.10}$$

Introducing time to see the effect of percentage changes on (1.8-1.10) gives the following:

$$FIDELITY-BANK(\mathrm{FB})=\left(t_1,t_2,t_3\right)\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}=\left(\alpha_1,\alpha_2,\alpha_3\right) \tag{1.11}$$

$$\mathrm{FIDELITY-BANK}(FB)'=\begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix} \tag{1.12}$$

$$ACCESS-BANK(\mathrm{AB})=\left(t_1,t_2,t_3\right)\begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix}=\left(\beta_1,\beta_2,\beta_3\right) \tag{1.13}$$

$$ACCESS-BANK\left(AB\right)'=\begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix} \tag{1.14}$$

$$FIDELITY-ACCESS-BANK-MERGED(\mathrm{FAM})=\left(t_1,t_2,t_3\right)\begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{pmatrix}=\left(\phi_1,\phi_2,\phi_3\right) \tag{1.15}$$

$$FIDELITY - ACCESS - MERGED\left(FAM\right)' = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix} \quad (1.16)$$

Combining (1.11-1.16) gives the following system of equations which will be used for the analysis of Fidelity, Access and their future merging independently, hence we have:

Fidelity Bank:

$$\begin{aligned} &a_{11}F_1 + a_{12}F_2 + a_{13}F_3 + \ldots + a_{1n}F_n = \alpha_1 \\ &a_{21}F_1 + a_{22}F_2 + a_{23}F_3 + \ldots + a_{1n}F_n = \alpha_1 \\ &\vdots \qquad\qquad\qquad\qquad\qquad \vdots \\ &a_{m1}F_1 + a_{m2}F_2 + a_{m3}F_3 + \ldots + a_{1n}F_n = \alpha_n \end{aligned} \quad (1.17)$$

Access Bank:

$$\begin{aligned} &b_{11}A_1 + b_{12}A_2 + b_{13}A_3 + \ldots + b_{1n}A_n = \beta_1 \\ &b_{21}A_1 + b_{22}A_2 + b_{23}A_3 + \ldots + b_{1n}A_n = \beta_1 \\ &\vdots \qquad\qquad\qquad\qquad\qquad \vdots \\ &b_{m1}A_1 + b_{m2}A_2 + b_{m3}A_3 + \ldots + b_{1n}A_n = \beta_n \end{aligned} \quad (1.18)$$

FIDELITY-ACCESS-BANKS-MERGED IN FUTURE:

$$\begin{aligned} &c_{11}FAM_1 + c_{12}FAM_2 + c_{13}FAM_3 + \ldots + c_{1n}FAM_n = \phi_1 \\ &c_{21}FAM_1 + c_{22}FAM_2 + c_{23}FAM_3 + \ldots + c_{1n}FAM_n = \phi_1 \\ &\vdots \qquad\qquad\qquad\qquad\qquad \vdots \\ &c_{m1}FAM_1 + c_{m2}FAM_2 + c_{m3}FAM_3 + \ldots + c_{1n}FAM_n = \phi_n \end{aligned} \quad (1.19)$$

Where $a_{ij}, i = 1,2,3\ldots m; j = 1,2,3\ldots n; b_{ij}, j = 1,2,3\ldots n; c_{ij}, i = 1,2,3\ldots \mathrm{m}, \mathrm{j} = 1,2,3\ldots \mathrm{n}$ are well known coefficient of the system and known scalars, and the unknown (variables).

3.1 Analysis and Results

This Section presents analyzed results whose methods are stated in Chapter three. Hence we have the following parameter values:

$Fidelity \ \ and \ \ Access \ bank: r = 0.03, \sigma = 0.25, k = 450$ and $t = 1$, Merged $Banks: k = 700$ which were implemented using Matlab programming software:

Table 1. 1: The value of Share price of Fidelity Bank, PLC According to trading days for European Call Option

Initial share price (S_0)	**The value of share prices when time $t=6$**	**The value of share prices when time $t=12$**
415	100	140
62	0.016	0.445
138	1.653	8.00
61	0.014	0.417
121	0.852	5.232
81	0.088	1.268
139	1.713	8.188
80	0.081	1.209
384	81.09	119

Table 1. 2: The value of Share price of Fidelity Bank, PLC According to trading days for European Put Option

Initial share price (S_0)	**The value of share prices when when time $t=6$**	**The value of share prices when time when time $t=12$**
415	407	413
62	52	53
138	130	133
61	51	52
121	113	116
81	72	73
139	131	134
80	71	72
384	376	382

Table 1.3: The value of Share price of Access Bank, PLC According to trading days for European Call Option

Initial share price (S_0)	**The value of share prices when when time** $t=6$	**The value of share prices when time when time** $t=12$
410	97	137
80	0.08	1.21
126	1.05	5.98
79	0.075	1.15
98	0.271	2.54
92	0.188	2.028
127	1.093	6.13
91	0.176	1.95
378	77.57	114.1

Table 1.4: The value of Share price of Access Bank, PLC According to trading days for European Put Option

Initial share price (S_0)	**The value of share prices when when time** $t=6$	**The value of share prices when time when time** $t=12$
410	402	408
80	71	72
126	118	121
79	70	71
98	90	92
92	84	85
127	119	122
91	83	84
378	370	376

Table 1.5: The value of Share price of Fidelity-Access Merged According to trading days for European Call Option

Initial share price (S_0)	**The value of share prices when time when time** $t=6$	**The value of share prices when time when time** $t=12$
825	449	491
142	1.89	8.76
264	24.93	50
140	1.77	8
219	12.51	31
173	4.72	16
266	25.60	51
171	4.48	15
762	389	432

Table 1.6: The value of Share price of Fidelity-Access Merged According to trading days for European Put Option

Initial share price (S_0)	**The value of share prices when time when time** $t=6$	**The value of share prices when time when time** $t=12$
825	817	823
142	134	137
264	256	261
140	132	135
219	211	216
173	165	169
266	258	263
171	163	167
762	754	760

3.2 Matrix Set up of Fidelity, ACCESS Banks and future merged Share prices:

$$FIDELITY-BANK(FB)=\begin{pmatrix}415 & 62 & 138\\ 61 & 121 & 81\\ 139 & 80 & 384\end{pmatrix}$$

$$ACCESS-BANK(AB)=\begin{pmatrix}410 & 80 & 126\\ 79 & 98 & 92\\ 127 & 91 & 378\end{pmatrix}$$

$$FIDELITY-ACCESS-MERGED(FAM)=\begin{pmatrix}825 & 142 & 264\\ 140 & 219 & 173\\ 266 & 171 & 762\end{pmatrix}$$

The effect 0f time on percentage changes are seen in appendix A

3.2.1 System of Equations on Share price of Fidelity, Access and Future Merged

Following our system of equations which will be used for the analysis of Fidelity, Access and their future merging independently, hence we have:

Fidelity Bank PLC:

$$\begin{aligned} & 415F_1 + 62F_2 + 138F_3 = 95.4000 \\ FBANK = {} & 61F_1 + 121F_2 + 81F_3 = 54.4000 \\ & 139F_1 + 80F_2 + 384F_3 = 145.200 \end{aligned}$$

$$F_1 = 0.1004,\ F_2 = 0.1978 \quad \text{and} \quad F_3 = 0.3006$$

Access Bank PLC:

$$\begin{aligned} & 410A_1 + 80A_2 + 126A_3 = 94.9000 \\ ABANK = {} & 79A_1 + 98A_2 + 92A_3 = 54.9000 \\ & 127A_1 + 91A_2 + 378A_3 = 144.4000 \end{aligned}$$

$$A_1 = 0.1006,\ A_2 = 0.1966 \quad \text{and} \quad A_3 = 0.3009$$

Fidelity-Access Merged:

$$\begin{aligned} & 825FAM_1 + 142FAM_1 + 264FAM_1 = 190.3000 \\ FAMERGED = {} & 140FAM_1 + 219FAM_1 + 173FAM_1 = 109.3000 \\ & 266FAM_1 + 171FAM_1 + 762FAM_1 = 289.6000 \end{aligned}$$

$$FAM_1 = 0.1005,\ FAM_2 = 0.1973 \quad \text{and} \quad FAM_3 = 0.3007$$

3.2.2 Analysis of the independent solutions from system of Linear Equations

Physically share prices from the solution of the systems connotes to profit of changes in share prices which may vary significantly in terms of growth. Hence, if the solutions are positives, then physical reality requires that the share prices are elements of real numbers. Below we state and prove proposition to show that our developed model satisfies this physical requirement.

Proposition 3.1: Let $F(t)$, $A(t)$ and $M(t)$ represents Fidelity, Access and Merged banks. Considering a long and short trading where investors will be faced with so many decisions due to share price changes. Supposed the independent solutions of the share prices grows at e^{3t} throughout the trading days. Assuming the trading of shares or adjustment of portfolio is considered to take place; where recovering the past trading of share prices by summing and integrating the function three times for three banks such that their constant of integration will be assumed to be volatility K_i, $i = 1, 2, 3$. Therefore, the function is defined as **:**

$$(i)\ \mathrm{F}(t) = \int \left(o.1004e^{3t} + 0.1978e^{3t} + 0.3006e^{3t}\right)dt$$

$$(ii)\ A(t) = \int \left(o.1006e^{3t} + 0.1966e^{3t} + 0.3009e^{3t}\right)dt$$

$$(iii)\ M(t) = \int \left(o.1005e^{3t} + 0.1973e^{3t} + 0.3007e^{3t}\right)dt$$

Proof: we want to show that the solution of the system obey some physical laws and follows exponential.

$$(i)\ \mathrm{F}(t) = \int \left(o.1004e^{3t} + 0.1978e^{3t} + 0.3006e^{3t}\right)dt = 0.1004\int e^{3t}dt + 0.1978\int e^{3t}dt + 0.3006\int e^{3t}dt$$

$$= 0.1004\frac{e^{3t}}{3} + 0.1978\frac{e^{3t}}{3} + 0.3006\frac{e^{3t}}{3} + K_1$$

i ntegrating for the second time gives as follows:

$$= 0.1004\frac{e^{3t}}{9} + 0.1978\frac{e^{3t}}{9} + 0.3006\frac{e^{3t}}{9} + \mathrm{K}_1\mathrm{t}$$

integrating for the third time gives as follows:

$$0.1004\frac{e^{3t}}{27} + 0.1978\frac{e^{3t}}{27} + 0.3006\frac{e^{3t}}{27} + \mathrm{K}_1\frac{t^2}{2} + \phi$$

Similarly

$$(ii)\ A(t) = \int \left(o.1006e^{3t} + 0.1966e^{3t} + 0.3009e^{3t}\right)dt = 0.1006\int e^{3t}dt + 0.1978\int e^{3t}dt + 0.3006\int e^{3t}dt$$

Integrating the above function gives: $0.1006\frac{e^{3t}}{3} + 0.1966\frac{e^{3t}}{3} + 0.3009\frac{e^{3t}}{3} + K_1$

Integrating for the second time gives as follows:

$$= 0.1006\frac{e^{3t}}{9} + 0.1966\frac{e^{3t}}{9} + 0.3009\frac{e^{3t}}{9} + K_2t$$

Integrating for the third time gives as follows:

$$0.1006\frac{e^{3t}}{27} + 0.1966\frac{e^{3t}}{27} + 0.3009\frac{e^{3t}}{27} + K_2\frac{t^2}{2} + \phi$$

Also

$$(iii)\ M(t)=\int\left(o.1005e^{3t}+0.1973e^{3t}+0.3007e^{3t}\right)dt=0.1005\int e^{3t}dt+0.1973\int e^{3t}dt+0.3007\int e^{3t}dt$$

integrating the above function gives: $0.1005\frac{e^{3t}}{3}+0.1973\frac{e^{3t}}{3}+0.3007\frac{e^{3t}}{3}+K_2$

integrating for the second time gives as follows:

$$=0.1005\frac{e^{3t}}{9}+0.1973\frac{e^{3t}}{9}+0.3007\frac{e^{3t}}{9}+K_2t$$

integrating for the third time gives as follows:

$$=0.1005\frac{e^{3t}}{27}+0.1973\frac{e^{3t}}{27}+0.3007\frac{e^{3t}}{27}+K_2\frac{t^2}{2}+\phi$$

where ϕ is constant of integration.
Hence the claim is true.

4.1 Discussion of Findings

Tables 1.1-1.4 shows the interpretations of Call and Put options with different maturity dates in terms of investment plans. Different maturity dates can give investor or the banks more flexibility to adjust positions based on changing market conditions. For example, if the banks are bullish on a stock in the long-term but bearish in the short-terms they could buy a Call option with a long-term maturity date while simultaneously selling a Call option with a shorter term. Also, options with different maturity dates can be used to hedge against potential losses.

Tables 1.5 and 1.6 shows the value of Fidelity and Access banks share prices as they merge in future. It can be seen from the variations of maturity days that increase in the maturity days increases the value of call option prices. Careful looking at the call option prices as they merge; one will understand that it is more profitable for the two banks to merge because the value of their assets will increase tremendously as seen above. This remark is encouraging in every investments because it is profit maximizing which will guide the management of banks, the ways of taking decisions based on the levels of their investments.

In subsection 3.2.1, which shows the solutions of system of linear equations for Fidelity, Access and Merged banks represents different trading strategies that investors could use to profit from changes in share prices. The three solutions also portray different levels of risk associated with investing in a particular stock or sector. In general, solving system of linear equations can help investors or management of Fidelity, Access banks gain insight into their investments.

5.1 Conclusion

This paper studied the framework of Black-Scholes model of European options on share price of Fidelity, Access and their future merged banks which gave closed form prices of Call and Put option prices with variations of maturity dates. More so, the share price of the independent banks were used to form a system of linear equations; which gave three different solutions according to the banks under considerations whose solutions represents different outcomes for share prices to analyze and predict for the purpose of investment plans for capital markets. To this end, we state and prove a proposition on solution of system of linear equations to show that our share price follows exponential series and obey some physical laws arising from financial markets.

Nevertheless, the considerations of three or more share prices of banks are highly recommended.

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Appendix

The Effect of time on percentage changes of Fidelity, Access and their future merging

Introducing time to see the effect of percentage changes on (3.3.1-3.3.3) gives the following:

$$FIDELITY-BANK(FB)=\left(0.1,0.2,0.3\right)\begin{pmatrix}415 & 62 & 138\\ 61 & 121 & 81\\ 139 & 80 & 384\end{pmatrix}=\left(95.4000,54.4000,145.200\right)$$

$$\text{FIDELITY}-\text{BANK}(FB)'=\begin{pmatrix}95.4000\\ 54.4000\\ 145.2000\end{pmatrix}$$

$$ACCESS-BANK(AB)=\left(0.1,0.2,0.3\right)\begin{pmatrix}410 & 80 & 126\\ 79 & 98 & 92\\ 127 & 91 & 378\end{pmatrix}=\left(94.9000,54.9000,144.4000\right)$$

$$ACCESS-BANK\left(AB\right)'=\begin{pmatrix}94.9000\\ 54.9000\\ 144.4000\end{pmatrix}$$

$$FIDELITY-ACCESS-MERGED(FAM)=\left(0.1,0.2,0.3\right)\begin{pmatrix}825 & 142 & 264\\ 140 & 219 & 173\\ 266 & 171 & 762\end{pmatrix}=\left(190.3000,109.3000,289.6000\right)$$

$$FIDELITY-ACCESS-MERGED\left(FAM\right)'=\begin{pmatrix}190.3000\\ 109.3000\\ 289.6000\end{pmatrix}$$

Appendix 2:Summary of solution of systems of linear equation

$$\begin{pmatrix}FIDELITY & ACCESS & MERGED\\ 0.1004 & 0.1006 & 0.1005\\ 0.1978 & 0.1966 & 0.1973\\ 0.3006 & 0.3009 & 0.3007\end{pmatrix}$$